Magnetic Field of a Helmholtz Coil

Introduction

A Helmholtz coil is a parallel pair of identical circular coils spaced one radius apart and wound so that the current flows through both coils in the same direction. This winding results in a uniform magnetic field between the coils with the primary component parallel to the axis of the two coils. The uniform field is the result of the sum of the two field components parallel to the axis of the coils and the difference between the components perpendicular to the same axis.

The purpose of the device is to allow scientists and engineers to perform experiments and tests that require a known ambient magnetic field. Helmholtz field generation can be static, time-varying DC, or AC, depending on the applications.

Applications include cancelling the earth's magnetic field for certain experiments; generating magnetic fields for determining magnetic shielding effectiveness or susceptibility of electronic equipment to magnetic fields; calibration of magnetometers and navigational equipment; and biomagnetic studies.

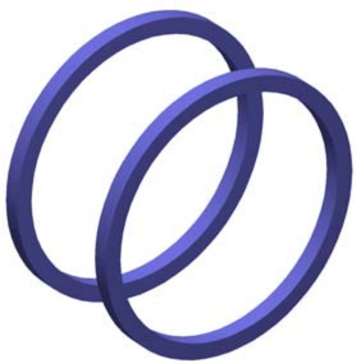


Figure 6-6: The Helmholtz coil consists of two coaxial circular coils, one radius apart along the axial direction. The coils carry parallel currents of equal magnitude.

Model Definition

The model is built using the 3D Magnetostatic application mode. The model geometry is shown in Figure 6-7.

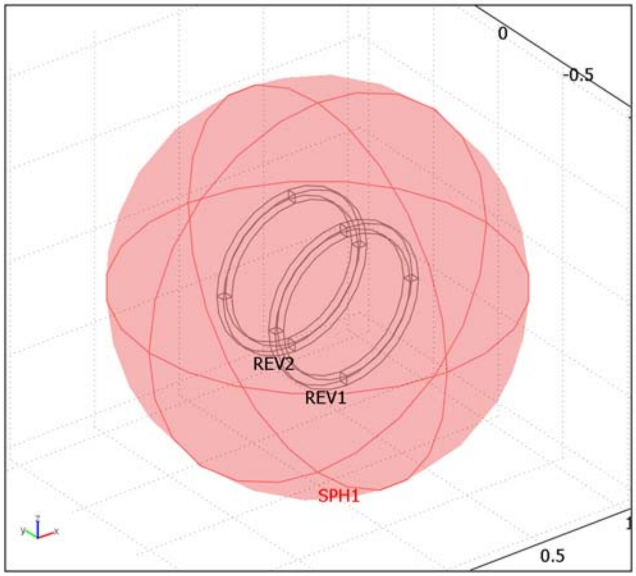


Figure 6-7: The model geometry.

DOMAIN EQUATIONS

Assuming static currents and fields, the magnetic vector potential **A** must satisfy the following equation:

$$\nabla \times (\mu^{-1} \nabla \times \mathbf{A}) = \mathbf{J}^e$$

where μ is the permeability, and $\mathbf{J}^e$ denotes the externally applied current density.

The relations between fields and potentials are given by

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{H} = \mu^{-1} \mathbf{B}$$

This model uses the following parameter value:

$$\mu = 4\pi \times 10^{-7} \qquad \text{(H/m)}$$

To avoid numerical instability, the application mode has an extra equation that sets the divergence of the **A** field to zero (gauge fixing). The external current density is zero except in the circular coils, where a current density of 1 A/m^2 is specified. This corresponds to a coil current of 2.5 mA. The currents are specified to be parallel for the two coils.

BOUNDARY CONDITIONS

The only boundary conditions that you need to specify is for the exterior boundary, that is, the spherical surface, where you apply conditions corresponding to zero magnetic flux:

$$\mathbf{n} \times \mathbf{A} = 0$$

Results and Discussion

Figure 6-8 shows the magnetic flux density between the coils. You can see that the flux is fairly uniform between the coils, except for the region close to the edges of the coil.

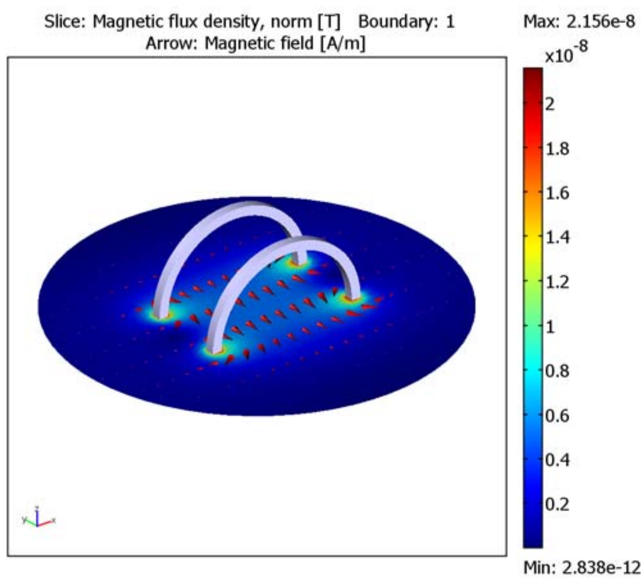


Figure 6-8: The surface color plot shows the magnetic flux density. The arrows indicate the magnetic field (H) strength and direction.

The main property of the Helmholtz coil is that the magnetic flux becomes uniform in a reasonably large region with a rather simple coil system. This is illustrated in Figure 6-9 (a), which shows a radial flux density profile for an axial position right between the coils. Figure 6-9 (b) shows an axial magnetic flux density profile. The model clearly demonstrates the highly uniform magnetic field obtained in a Helmholtz coil.

In the absence of any test object, this model is fully axisymmetric and could be implemented as a 2D axisymmetric model, which would be much less computationally demanding. However, this full 3D model has the advantage that a non axisymmetric test object could be included in the analysis as a slight modification of the model.

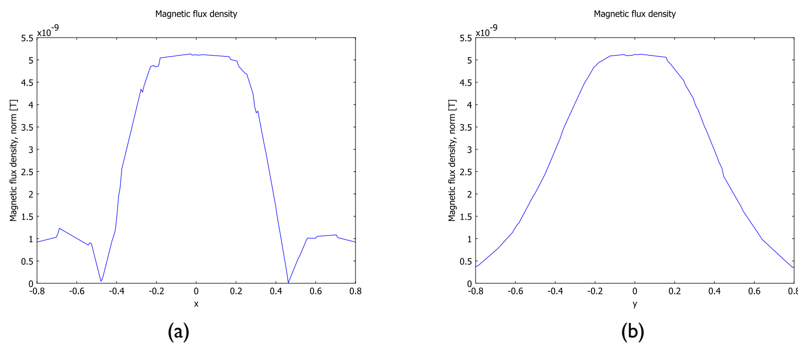


Figure 6-9: The magnetic flux density profile. In the graph at the left, the profile is taken along a radial cross sectional line through the axis right between the coils. In the graph on the right, the profile is taken along the axis. The high degree of uniformity is clearly shown.

Model Library path: `AC/DC_Module/Electrical_Components/helmholtz_coil`

Modeling Using the Graphical User Interface

MODEL NAVIGATOR

1. Select **3D** in the Space dimension list.
2. In the list of application modes, select **AC/DC Module>Statics>Magnetostatics**.
3. From the **Element** list choose **Vector - Quadratic**.
4. Click **OK**.

OPTIONS AND SETTINGS

From the **Options** menu, open the **Constants** dialog box and enter the following names and expressions.

NAME	EXPRESSION	DESCRIPTION
J0	1	Current density in coil (A/m^2)

GEOMETRY MODELING

The coils are made from squares in a 2D work plane, which you then revolve into a 3D geometry.

1 From the **Draw** menu open the **Work Plane Settings** dialog box. Click **OK** to obtain the default work plane in the *x*-*y* plane.

2 In the 2D geometry, choose **Options>Axes/Grid Settings** and specify the settings according to the following table (you must clear the **Auto** check box on the **Grid** page to be able to enter the grid settings)

AXIS		GRID	
x min	-0.6	x spacing	0.4
x max	-0.2	Extra x	
y min	-0.4	y spacing	0.2
y max	0.4	Extra y	-0.225 -0.175 0.175 0.225

3 Draw a square with corners at (–0.425, 0.175) and (–0.375, 0.225).

4 Draw another square with corners at (–0.425, –0.225) and (–0.375, –0.175).

5 Select both squares and use the **Revolve** dialog box available from the **Draw** menu to revolve into the 3D geometry.

6 Finally add a sphere with radius 1 and center at the origin to the 3D geometry.

PHYSICS SETTINGS

Boundary Conditions

Use the default **Magnetic insulation** boundary condition.

Subdomain Settings

Apply the current in the coils as a given source current. Open the **Subdomain Settings** dialog box from the **Physics** menu and enter the source current according to the following table.

SUBDOMAIN	1	2, 3
$\mathbf{J}^e$	`0 0 0`	`-J0*z/sqrt(x^2+z^2) 0 J0*x/sqrt(x^2+z^2)`

MESH GENERATION

1 Open the **Free Mesh Parameters** dialog box and select **Coarser** from the **Predefined mesh sizes** list on the **General** tab. Click the **Subdomain** tab, select the subdomains 2 and 3, and set the **Maximum element size** to `0.05`.

2 Click **Remesh** to generate the mesh, then click **OK**.

COMPUTING THE SOLUTION

Using the geometric multigrid solver, it is possible to use the linear order element combination for a coarse solution and solve for the quadratic elements. The linear solution is then used in the preconditioning step.

1 Open the **Solver Parameters** dialog box from the **Solve** menu.

2 Select **Geometric multigrid** from the **Linear system solver** list.

3 Click the **Settings** button. In the dialog box that appears, make sure that **Linear system solver** is selected in the field to the left. Select **Low element order first** from the **Hierarchy generation method** list.

4 All other settings can be left at their default values. For details on the default settings, see "Solving Large 3D Problems" on page 67 of the *AC/DC Module User's Guide*.

5 Click **OK** to close the **Linear System Solver Settings** dialog.

6 Click **OK** to close the **Solver Parameters** dialog box.

7 Click the **Solve** button.

POSTPROCESSING AND VISUALIZATION

1 Suppress the surface of the sphere, that is, boundaries 1, 2, 3, 4, 21, 22, 31, and 32 using the **Suppress Boundaries** dialog box in the **Options** menu.

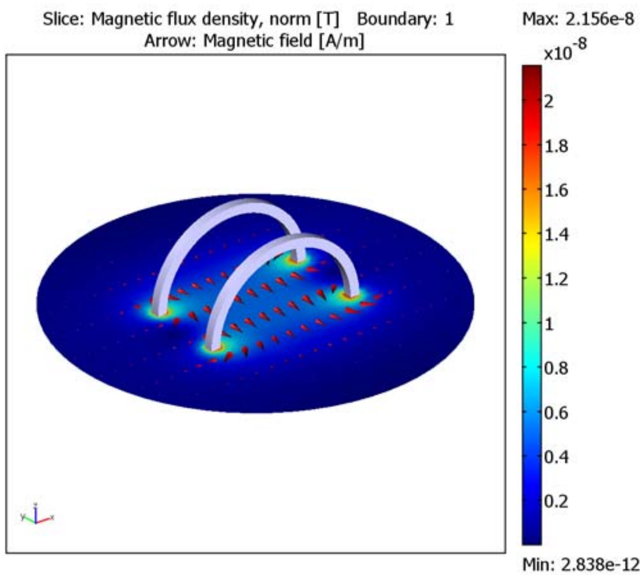
2 On the **General** page of the **Plot Parameters** dialog box select the **Slice**, **Boundary**, and **Arrow** check boxes and clear the **Geometry edges** check box.

3 On the **Slice** page use the default **Slice data**, which is **magnetic flux density, norm**. Set the number of **x levels** to `0` and the number of **z levels** to `1`.

4 On the **Boundary** page set **Boundary data** to `1` and select a **Uniform color** of your choice.

5 On the **Arrow** page select **Magnetic field** as **Arrow data**, set the **x points** to `24`, the **y points** to `10`, and the **z points** to `1`. Enter `0.5` as **Scale factor**.

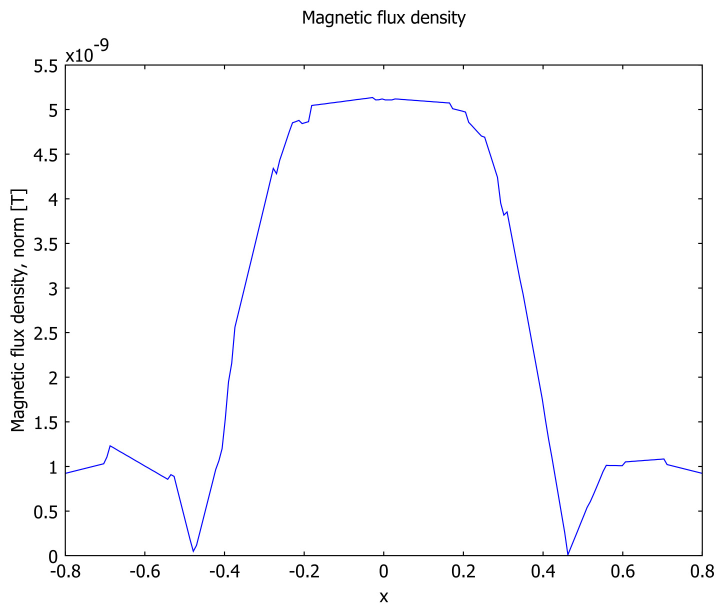
6 To add some lighting to the plot open the **Visualization/Selection Settings** dialog box from the **Options** menu and select **Scenelight** on the **Lighting** page. Disable lights 1 and 3.



You can visualize the magnetic flux density profile using cross-section plots.

1 Open the **Cross-Section Plot Parameters** dialog box. On the **General** page click the **Title/Axis** button and set the title to `Magnetic flux density`.

2 On the **Line** tab use the default **y-axis data** which is **Magnetic flux density, norm**. Set the **x-axis data** to `x` and set **x0** to `-0.8` and **x1** to `0.8`. Click **Apply** to make a plot of the magnetic flux density along a line through the axis right between the coils.



To make a plot in the other direction change the **x-axis data** to y and set **x0** to 0, **x1** to 0, **y0** to -0.8 and **y1** to 0.8.